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ABSTRACT

Vibration tests consisting of heel impacts by a person, shaker impacts, and random shaker excitations were carried out on an 8.5 × 8.8 m steel joist-concrete slab floor sample. Structural variations to the floor sample included restrained and free edge conditions, two concrete slab thicknesses, several joist stiffnesses, and transverse bracing of the steel joists. The lowest four to six natural frequencies, associated mode shapes and modal damping values were determined for various structural configurations. Modal damping of the floor sample was significantly increased by the presence of one or more persons on the floor, whereas structural modifications had relatively little effect. Predicted and measured initial peak accelerations due to heel impact and fundamental floor frequencies were also compared.

INTRODUCTION

As floor systems in buildings become structurally more efficient, they also tend to become more responsive to common dynamic loadings such as people walking, dancing or jumping, vehicular traffic, and machinery. The main factors contributing to this increased vibration sensitivity are reductions in mass, stiffness, and damping. Since these parameters dominate the dynamic behaviour of structural systems, a better understanding and quantitative evaluation of them will allow the prevention or alleviation of objectionable vibrations.

An opportunity to study these parameters arose when a full-scale floor model became available for dynamic testing. The objective of the tests was to measure the dynamic properties of the composite steel joist-concrete slab floor sample and to ascertain how these properties were affected by structural modifications to the floor sample. Particular emphasis was placed on the evaluation of damping, as this parameter cannot as yet be calculated.

Some aspects of this investigation have been described in an earlier paper.[1] This report is more extensive in its presentation of modal frequencies, mode shapes, and modal damping ratios.

EXPERIMENTAL PROCEDURE

The 8.8 × 8.5 m floor sample comprised a nominally 75 mm thick concrete deck,* composite with six 457 mm deep steel joists. The joists were

*Since the measured thickness from concrete core samples was 81 mm, the slab is henceforth considered to be 81 mm thick.

supported on steel girders, which in turn framed into four corner steel columns. The edges of the sample were also bounded by steel beams (see Figure 1 for details).

Instrumentation for measuring the dynamic response of the floor consisted of accelerometers located at the grid points shown in Figure 2. The signals from the accelerometers were filtered and amplified before being recorded on FM tape recorders. A portable oscilloscope and spectrum analyzer were used to monitor the signals during the experiments. At five points on the floor (labelled A to E in Figure 2), dynamic loads were applied from random shaker excitation, shaker impact, and heel impacts by a person.

Modifications to the original floor sample consisted of: 1) end beams supported by screw jacks of a type commonly used to support floors in the basements of houses, with supported or unsupported edge beams (positions shown in Figure 1); 2) suspended ceiling and gypsum board ceiling on the underside of the floor; 3) diagonal steel bracing between the joists; 4) a 152 × 229 mm concrete stiffening beam normal to the joists; 5) steel-neoprene-steel sandwich strips (6 × 76 mm - 7 × 75 mm - 6 × 76 mm) bolted to the lower chord of the joists; and 6) a 25 mm concrete topping added to the slab.

Floor responses produced by random and impact excitations were analyzed on a Hewlett-Packard 5423A spectrum analyzer to obtain natural frequencies and phase between transducer locations. Mode shapes were drawn from spectral amplitudes and phase. Modal damping ratios were found by two methods: 1) a curve-fitting procedure of the spectrum analyzer applied to the spectral peaks of the Fourier spectrum of the floor response; and 2) for shaker and heel impact signals, the free vibration decay curves were band-pass filtered around the natural frequency and logarithmic decrements computed. Examples of Fourier spectra for shaker random and impact excitation are shown in Figures 3 and 4; time records of free vibration decay curves on linear scales are shown in Figures 5 and 6, and on linear and logarithmic scales in Figure 7a and b. The vibration decay curves were obtained by adding the signals from symmetrically-placed transducers for mode 1, a symmetrical mode, and subtracting them for mode 2, an anti-symmetric mode. This eliminated anti-symmetric modes almost coincident in frequency from the former, and symmetric modes from the latter. Other undesirable modal components were removed by suitable band-pass filtering.

RESULTS

Natural Frequencies and Mode Shapes

Tables 1 and 2 give natural frequencies and associated mode shapes along the joists and across the joists, as depicted in the table header. Natural frequencies with asterisks identify those modes whose shapes could not be positively identified from the data. They have been fitted into the tables by considering their position in the sequence of modal frequencies.

For some of the higher modes in Table 1, relatively large modal amplitudes were measured on the end or edge beams, contrary to what might be expected for supported end and edge conditions. Although some small

amplitudes will always be present due to elastic shortening of the jacks, the relatively large amplitudes are thought to be due to local separation of the slab from the top flange of the end and edge beams. It is therefore questionable whether fully-supported end and edge conditions apply to the higher modes.

Table 2 presents natural frequencies and mode shapes for the floor sample with supported or unsupported end and edge beams. As a result of reduced perimeter restraint, there were more modes with natural frequencies below 22 Hz for the unsupported than for the fully-supported conditions. It was difficult to obtain the mode shape for mode 3 because the excitation point was near a node, except for Run No. 111, with excitation at location E.

Measured mode shapes of the original floor sample are presented in Figures 8 to 10 for various end and edge support conditions. The plotted points are transducer locations for which modal amplitudes were determined. Extrapolated portions of the mode shapes are shown by dashed lines. For supported end and edge conditions, modes with successively higher natural frequencies contained an increasing number of nearly sinusoidal lobes, occurring first across the joists, and then along the joists, as shown in Figure 9.

The results of Tables 1 and 2 and Figures 8 to 10 showed the following trends:

1) Supporting the end beams primarily increased the natural frequencies of modes with nodal lines perpendicular to the joists. The largest increase in frequency occurred in the fundamental and the fifth mode.
2) Supporting the edge beams had little effect on the frequency of the fundamental mode, but substantially increased the frequencies of the second mode and higher modes with nodal lines parallel to the joists.
3) As in 2), the addition of stiffening elements perpendicular to the joists primarily raised the frequencies of those modes with nodal lines parallel to the joists.
4) The addition of 25 mm of concrete to the slab reduced the natural frequency of all modes by at least 5%.

The above observations generally agree with what might be expected from principles of vibration theory pertaining to plates.

A small decrease in the first and second modal frequencies is discernible in Table 2, between what were originally thought to be identical runs, Run 5 and Runs 8, 9 and 10. These differences can be attributed to slight changes in the perimeter conditions of the floor sample. It is thought that jacking operations on the end and edge beams and the joists caused local concrete cracking and loss of bond between the steel beams and the concrete slab. Further separation between the concrete slab and the steel beams may also have occurred as the concrete dried and shrank. The resulting loss of bond would slightly decrease the end and edge stiffness of the floor sample and result in lower modal frequencies.

Damping of the Floor Sample

Modal damping ratios for shaker impact and heel impact response were obtained from the free vibration decay using the relationship:

$$\xi = \frac{1}{2\pi n} \log_e \frac{X_m}{X_{m+n}} = \frac{1}{2\pi n} \delta \qquad (1)$$

where: ξ = modal damping ratio, fraction of critical;
X_m = amplitude of the mth peak;
X_{m+n} = amplitude of the peak n cycles later;
δ = logarithmic decrement.

Vibration decays for the first two modes are shown in Figures 5, 6 and 7. A convenient display of the decay in logarithmic amplitude, shown in Figure 7b, provided a simple method for calculating the logarithmic decrement (δ) by fitting a straight line through the peaks. It also afforded the opportunity to observe any changes in the logarithmic decrement with progressively smaller amplitudes; i.e., to detect amplitude-dependent damping. The logarithmic decrement for each mode and the modal damping ratio were computed for the first 20 cycles except where otherwise noted.

Modal damping ratios for shaker impact and shaker random excitation were also obtained using the curve-fitting algorithm for Fourier spectrum peaks provided by the Hewlett-Packard 5323A Structural Dynamics Analyzer. Some results using both procedures are presented in Table 3 for different configurations of the floor sample. In general, comparable modal damping ratios were obtained using both methods. On occasion, however, the agreement was less than satisfactory (Run 183, Mode 2). This may be due to inadequate spectral resolution in the Fourier spectrum or inappropriate placement of the cursors for bounding the spectral peak of the curve-fitting algorithm on the spectrum analyzer.

Damping contributed by people: The heel impact has long been a favourite source of floor excitation because of its simplicity and its resemblance to other dynamic loadings on floors such as walking and jumping.[2,3] However, this test requires the presence of at least one person on the floor. A comparison of modal damping ratios derived from tests demonstrated that a significant increase in modal damping can result when people are placed on a floor that has inherently little damping. Table 4 illustrates this effect by comparing modal damping ratios derived from heel impact with those from shaker impact tests with and without people on the floor. The effect of one, two or three persons on the measured damping ratios is demonstrated there.

The location of a person on the floor also affected the amount of increase in the total measured damping ratio. The results for the heel impact test, Run No. 155, in Tables 4 and 5 show that the modal damping ratio for the fundamental mode decreased as the person moved from location A, the centre of the floor, to location D. This decrease in damping is attributed to decreases in modal amplitude with location. In Table 5 the modal damping ratios measured at locations B, C and D were adjusted by the square of the ratio of modal amplitudes relative to location A, resulting in damping ratios for persons as if they were standing at location A. The

mathematical basis for this adjustment is presented in Figure 11 and is discussed in a later section. For the floor sample with supported end and edge beams, each person standing at position A (the centre of the floor sample), contributes between 0.80 and 1.00% of critical to the damping ratio for the fundamental mode of the floor sample. Aspects of the mathematical modelling of damping due to people are also treated in a later section.

The amount of modal damping also varied with the individual. Although no systematic study was done in this series of tests, there are indications that the damping contributed by people varied from person to person, depending on weight and physique.

Changes in damping due to floor modifications: Major floor modifications can be expected to result in different modal damping ratios as new sources of damping are introduced and others diminish. The following modifications to the floor sample were investigated: end beams and edge beams fully supported; suspended ceiling on underside of floor; gypsum board ceiling on underside of floor; steel-neoprene damping strips bolted to bottom chord of the joists. The results are summarized in Table 6.

For assessing the influence of various floor modifications, modal damping ratios were determined from shaker impact tests. Heel impact tests could not be utilized since they contained the additional damping of the person doing the test. Modal damping was calculated from the logarithmic decrement of the free vibration decay curve, as it was considered to give a more reliable estimate than applying the curve-fitting algorithm of the spectrum analyzer to the Fourier spectrum peak.

The results presented in Table 6 show that for the floor without modifications, and for all combinations of supported and unsupported end and edge beams, the damping ratio for the fundamental mode (ξ_s) was 0.37 or 0.4% of critical. The addition of a gypsum board ceiling raised this value to 0.44%. A suspended ceiling with simulated lights increased the damping ratio to a greater degree, to 0.62% of critical for unsupported end and edge beams and 0.71% for supported end beams. The largest change in damping ratio occurred with the addition of the steel-neoprene sandwich strips, which for supported end beams resulted in 1.0% modal damping ratio, and for the unsupported beams gave 0.76% of critical. A larger modal mass as a result of end and edge beam movement without a commensurate increase in damping mechanisms may have been responsible for the smaller damping ratios for the unsupported end and edge beams. It must also be noted that a substantial increase in stiffness occurred with the application of the sandwich damping strips, as can be seen by the increase in the fundamental frequency from 6.4 to 7.8 Hz for the supported end beams.

Response to Heel Impact

A number of vibration criteria for performance of floors incorporate the initial peak amplitude of floor acceleration to an actual or mathematical representation of a heel impact. This applies to the criteria presented by Lenzen[2] and the criteria in the appendix of CSA Steel Standard.[3] For floors with fundamental frequency less than about 10 Hz, the initial peak acceleration amplitude (a_0) in units of gravity (g) can be calculated from:[4]

$$a_0 = \frac{2\pi f\ I}{gM_E}\ (0.9) \qquad (2)$$

where f = fundamental frequency in Hz, I = equivalent impulse of heel impact = 67 Ns, and M_E = equivalent mass in kg. An approximate equivalent mass M_J for a simply-supported joist floor can be obtained by assuming a tributary strip 60 t_c wide,[3,4] giving:

$$M_J = (0.40)\ L\ (60)\ t_c(t_c + 1)/g \qquad (3)$$

An examination of the fundamental mode shapes of the floor sample indicates, however, that the entire width of the sample should be used for the calculation of the equivalent mass rather than 60 t_c, since the entire sample width contributes to a half-sine wave. If one sums the elemental inertias in proportion to the first mode shape, then the equivalent inertial mass (M_I) is obtained:

$$M_I = \rho \int_0^a \int_0^b \sin x \sin y\ dx\ dy \qquad (4)$$

$$= 0.405\ \rho ab$$

= 0.405 times total mass of the floor sample

where ρ = mass density per unit area, and a,b = plan dimensions of the floor.

Calculated and measured peak initial acceleration amplitudes (a_0) are given in Table 7. The measured values shown were derived from averaging the results of four successive heel impacts. Calculated values of a_0 for locations B, C and D were obtained by adjusting the results of Equation (2) in proportion to the ratio of modal amplitudes at these locations, and those at location A, as is consistent with the properties of modal force presented in Figure 11.

The results in Table 7 show that the use of equivalent inertial mass (M_I) from Equation (4) for Run No. 33 gives excellent agreement between measurements and calculations. For the other two floor configurations (Run Nos. 155 and 190), the measured values are somewhat larger than the calculated ones. The latter two runs were performed on the floor modified by the transverse concrete beam, and the response to impulsive loads may be somewhat different from that assumed in the simple spring-mass model implied by Equation (2). On the other hand, use of M_J of Equation (3), the equivalent mass with a strip width of 60 t_c, gives peak accelerations that are considerably larger than the measured ones. This is perhaps not surprising, since the behaviour of the floor sample is obviously closer to the assumptions inherent in the derivation of the equivalent inertial mass (M_I) than those of the approximate equivalent mass (M_J).

The measured initial peak acceleration amplitudes for the unsupported end and edge beams (Run Nos. 8 to 11) in Table 7 are considerably larger than for the comparable cases with supported beams (Run No. 33). There is also a large discrepancy in a_0 derived from using the equivalent masses M_J and M_I for the unsupported beams of Run Nos. 8 to 11. This would indicate

that the method of calculating impulse response for floors with flexible support beams needs to be investigated further.

MATHEMATICAL MODELLING OF VIBRATING FLOORS

Since the theoretical predictions of initial peak acceleration a_0 have displayed some apparent inconsistencies, a closer examination of the basic assumptions in the mathematical modelling process appears warranted.

The main parameters in the vibration system (i.e., mass, damping, and natural frequency) will subsequently be discussed. In principle, the dynamic response of a floor system to an impact can be obtained by the method of modal superposition, as is possible for any vibrating linear elastic system. Details of the method can be found in standard textbooks on vibrations.[5] A summary of modal properties for a simple beam structure is presented in Figure 11.

Mass

Modal mass and modal damping are functions of the square of the mode shape (Ψ, Figure 11). This is in contrast to equivalent inertial mass in Equation (4), which is a function of the mode shape only to the power of 1. For a simply-supported plate, the modal mass of the fundamental mode equals 0.250 M, whereas the equivalent inertial mass (M_I) according to Equation (4) is 0.405 M (M is the total mass of the plate). Modal mass is the mathematically correct form for solving vibration problems as described by the equation of motion in Figure 11. Equivalent inertial mass arises from the empirical formulation of the impulse response of a system having distributed mass, and is only applicable to the calculation of peak impulse response. It cannot be used as the mass for the determination of natural frequency.

The accuracy in predicting a_0 for the floor sample using equivalent inertial mass (M_I) seems to depend on the end and edge conditions of the floor (Table 7). Considering the inherent variability of the technique, agreement between the calculated and measured values of a_0 for the supported end and edge conditions was excellent. For unsupported end and edge beams, however, the equivalent inertial mass model seems to be unsuitable, as is evident from the large difference between predicted and measured results. A more consistent formulation is needed, but this is beyond the scope of this report.

Damping

As was shown before, the variations in damping ratios for the fundamental mode with changing position of the person can be assessed using the mathematical model shown in Figure 11. However, this mathematical model has to be viewed somewhat as an artifact, since the person standing on the floor is surely not well represented by a single dashpot connected to the ground. A realistic representation of a person would require the use of a multi-degree-of-freedom system with many masses and stiffnesses, and damping dashpots not connected to the ground. The consideration of such a model is, however, beyond the scope of this report. Despite the apparent shortcomings

on a physical basis, the simple dashpot model gives answers that are in general agreement with the observed values (Table 5).

Fundamental Natural Frequency

The fundamental natural frequency of the floor is another important parameter in assessing the vibrational behaviour of a floor system. For a simply-supported joist floor, the frequency in Hz can be calculated from:[4]

$$f = 156 \sqrt{\frac{EI}{wL4}} \qquad (5)$$

where E = Young's modulus (MPa), I = moment of inertia (mm^4), w = loading per unit length (N/mm) and L = span length (mm). The fundamental frequency of the floor sample was calculated assuming fully composite action between the joist and the concrete slab and a tributary floor area equal to the joist spacing. Comparisons of calculated and measured frequencies given in Table 8 show excellent agreement for the cases where end and edge beams were supported.

For unsupported end and edge beams, the fundamental frequency (f) of the floor sample can be estimated by Dunkerly's approximation:

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} \qquad (6)$$

where f_1 = fundamental frequency of floor with edge and end beams supported, and f_2 = fundamental frequency of unsupported end beams. The calculation of f_2 requires some assumptions regarding the composite nature of the steel beam-concrete slab system, and the size of its tributary area. It was assumed here that one quarter of the floor slab contributes to both the mass and the stiffness of the end beam element. For the two cases, fully composite and noncomposite, the following frequencies were obtained for the 81 mm slab:

$$f_2 \text{ (noncomposite)} = 6.2 \text{ Hz}$$

$$f_2 \text{ (composite)} = 10.6 \text{ Hz} .$$

Substitution of f_2 into Equation (4) gives:

$$f = 4.7 \text{ Hz (noncomposite)}$$

and

$$f = 5.5 \text{ Hz (composite)} .$$

The comparison with measured values for other floor configurations is presented in Table 8. The composite assumption for the end beam gives better agreement with measurements than the noncomposite one. The effect of the transverse beam on the frequencies of the fundamental mode in Table 8 was ignored.

DISCUSSION

The study of the dynamic properties of the floor sample provided an opportunity to assess the influence of various parameters that cannot usually be investigated on a construction site or in completed and occupied structures. However, the floor sample suffers from one major disadvantage in that it does not represent an actual floor system. Real floors extend over several bays in both directions and contain many more sources of stiffness and damping such as walls, partitions and other supporting beams. These additional sources of stiffness can produce small to major changes in mode shapes and natural frequencies and thus result in changes to the vibration response of the floor to dynamic loading. Modal damping ratios obtained for people and for various structural modifications of the floor should therefore not be directly employed in other situations without appropriate verification of their applicability.

SUMMARY AND CONCLUSIONS

The dynamic tests on a full scale laboratory floor sample of composite steel joist-concrete slab construction indicated the following:

1. Mode shapes and natural frequencies can be evaluated satisfactorily by exciting the floor system with either heel impact, shaker impact, or shaker random excitation. Several points of excitation are needed if a complete set of modal properties is desired.
2. Modal damping ratios can be evaluated from either the logarithmic decrement method of free vibration decay from impact, or from the half-power bandwidth or curve-fitting methods applied to Fourier spectrum peaks derived from the floor response to random excitations or impacts. The presence of people on the floor affects the measured modal damping ratio, especially for floors with inherently low damping.
3. The contribution of people to modal damping and the amplitude of the initial peak acceleration to heel impact are both dependent on the location of the persons on the floor and the point of excitation. Corrections for these two effects can be made from a knowledge of the mode shape of the floor slab.
4. Significant differences were found between the calculated equivalent inertial mass (M_I) and the calculated equivalent mass (M_J) of the floor sample. Good agreement was obtained between calculated and measured values of the initial peak acceleration a_0 to a heel impact, for the case of supported end and edge beams, when the equivalent inertial mass was used. This agreement is ascribed to the similarity between the actual mode shape of the floor sample and the shape implied by the inertial mass model. On the other hand, the agreement was poor for unsupported edge and end beams. This would indicate that the inertial mass model is inappropriate for the calculation of initial peak acceleration for the floor sample with unsupported end and edge beams.
5. Addition of a suspended ceiling or a gypsum board ceiling raised the damping ratio of the fundamental mode from 0.4% to a maximum of 0.71% of critical. Although this represents a substantial percentage increase, it is only a relatively small increase in absolute terms. The most significant increase in the damping ratio of the fundamental mode occurred when the steel-neoprene sandwich strips were bolted to the

bottom chord of the joists. For the case with supported end and edge beams, a modal damping ratio of 1% of critical was obtained.

6. The agreement between measured and calculated fundamental frequency of the floor sample was good for cases where both end and edge beams were either supported or unsupported. For the latter case, the assumption of full composite action was required in calculating the frequency of the end beams. Assuming non-composite action for the end beams gave estimates of fundamental frequency that were too low.

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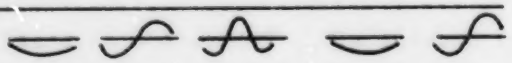
TABLE 1

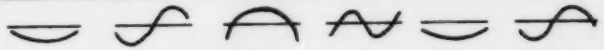
Modal Frequencies for Floor Sample With Supported End and Edge Beams

		Mode No.	1	2	3	4	5
Mode Shape	Across joist						
	Along joist						
	Floor Conditions	Run No.	Modal Frequencies, Hz				
	81 mm slab	29 30	6.50	9.60	15.75*	17.5	22.0
	81 mm slab with 15 × 23 mm transverse concrete beam	159 161	6.75	13.25 13.50	20.62	17.12	
	106 mm slab with 15 × 23 mm transverse concrete beam	181	6.25	11.6			

*Mode shape could not be identified with certainty from available data.

TABLE 2

Modal Frequencies of Floor Sample With Supported or Unsupported End and Edge Beams

		Mode No.	1	2	3	4	5	6
Mode Shape	Across joist							
	Along joist							
Floor Condition	End, Edge Beam Support Conditions**	Run No.	Modal Frequencies, Hz					
81 mm slab	U, U	5	5.47	8.98	12.8*	16.4	11.7	22.0
		8, 9, 10	5.43	8.79				
81 mm slab	S, U	16-19	6.4	8.9	12.8*	16.4	17.6	21.9*
81 mm slab	S, U	111	6.35	8.79	12.70	16.1	17.2	22.0
81 mm slab	U, U	146	5.4	8.8	12.6*	16.1	11.4	
81 mm slab with transverse concrete beam	U, U	147	5.6	9.8				
81 mm slab with transverse concrete beam	U, S	152	5.8	11.5				
106 mm slab with transverse concrete beam	U, U	166	5.3	9.2				
		169						

*Mode shape could not be identified with certainty from available data.
**U - unsupported;
S - supported.

TABLE 3

Modal Damping Ratios of Floor Sample

Floor Condition	End, Edge Beam Support Conditions*	Run No.	Mode No.	Modal Frequency, Hz	Modal Damping Ratio, ξ, % Critical		
					From Logarithmic Decrement, Shaker Impact	Curve-Fitting Algorithm of Spectrum Analyzer	
						Shaker Impact	Shaker Random
81 mm slab with transverse concrete beam	S, S	159	1	6.72	0.48	0.45	
		161	1	6.72	0.46	0.46	
			2	13.50	0.81	0.88	
81 mm slab	U, U	6	1	5.49			0.32
							0.39
							0.25
			2	8.93			
							0.96
							0.91
							0.88
			4	16.35			
							0.40
							0.58
							0.49
			5	11.66			
							0.48
							0.45
106 mm slab with transverse concrete beam	S, S	182	1	6.25	0.55	0.56	
		183	1	6.25	0.50	0.56	
			2	11.62	1.06	0.82	

*U - unsupported;
S - supported.

TABLE 4

Influence of Persons on Modal Damping Ratios of Floor Sample, End and Edge Beams Supported

Modal Frequency, Hz					Total Damping Ratio, % of Critical		Difference From Bare Floor	
Mode 1	Mode 2	Run No.	Floor Condition	Type of Excitation and Location	Mode 1	Mode 2	Mode 1	Mode 2
6.72	13.5	160	81 mm slab with transverse concrete beam	Shaker impact at A, bare floor	0.46	0.81		
				3 persons near A	3.39		2.93	
6.72	13.5	155	81 mm slab with transverse concrete beam	Heel impact at A	1.36		0.91	
				B	1.02	1.33	0.57	0.52
				C	1.03		0.58	
				D	0.94	1.21	0.49	0.40
6.25	11.62	182	106 mm slab with transverse concrete beam	Shaker impact at A, bare floor	0.55			
				1 person near A	1.40		0.85	
				2 persons near A	2.25		1.70	

TABLE 5

Damping Ratios for Fundamental Mode With One Person on Floor, End and Edge Beams Supported

Floor Condition	Run No.	Heel Impact at	Damping Ratio, ξ, % of Critical: Total	Comparable Bare Floor (Run No.)	Total Minus Bare Floor	Adjusted for Mode Shape Relative to Location A	Average for 1 Person at Location A
81 mm slab	77	A	1.19	0.37(76)	0.82	0.82	0.80
		B	0.85		0.48	0.79	
		C	0.82		0.45	0.81	
		D	0.72		0.35	0.77	
81 mm slab with suspended ceiling plus "lights"	93	A	1.67	0.71(92)	0.96	0.96	0.80
		B	1.35		0.64	1.05	
		C	1.17		0.46	0.83	
		D	1.06		0.35	0.77	
81 mm slab	155	A	1.36	0.46(160)	0.91	0.91	0.99
		B	1.02		0.57	0.93	
		C	1.03		0.58	1.04	
		D	0.94		0.49	1.07	

TABLE 6A

Damping Ratios (ξ, % of Critical) of Fundamental Mode for Various Floor Modifications, 81 mm Slab Unsupported End and Edge Beams

		Shaker Impact				Heel Impact		Shaker Random			
		From Logarithmic Decrement		Curve-Fitting Algorithm of Specturm Analyzer		From Logarithmic Decrement		Half-Power Bandwidth*		Curve-Fitting Algorithm of Spectrum Analyzer	
Floor Condition	Natural Frequency, Hz	Record No., Location of Excitation	ξ_S	Record No., Location of Excitation	ξ_{SP}	Record No., Location of Excitation	ξ_H	Record No., Location of Excitation	ξ_W	Record No., Location of Excitation	ξ_{WP}
a) no modifications	5.41		(0.4)**			135, A	0.91	146, E	0.37		
b) suspended ceiling with "lights"	5.30	99, E	0.62	99, E	0.99 0.91	98, A B C D	1.10 0.90 0.88 0.85	97, E	0.47	97, E	0.68
c) gypsum board ceiling	5.38	104, E	0.44	104, E		103, A B C D	0.98 0.97 0.88 0.81	100, E	0.38	100, E	0.44
d) steel-neoprene damping strips	5.8 double 6.2 peak	121, E	0.76	121, E	1.07	119, A	0.97	118, E	0.65 0.77	118, E	0.39

*Analysis bandwidth = 0.060 Hz.
**Extrapolated from comparisons between other cases of shaker impact and heel impact tests.

TABLE 6B

Damping Ratios (ξ, % of Critical) of Fundamental Mode for Various Floor Modifications, 81 mm Slab Supported End Beams, Unsupported Edge Beams

		Shaker Impact				Heel Impact		Shaker Random			
		From Logarithmic Decrement		Curve Fitting Algorithm of Spectrum Analyzer		From Logarithmic Decrement		Half-Power Bandwidth*		Curve Fitting Algorithm of Spectrum Analyzer	
Floor Condition	Natural Frequency, Hz	Record No., Location of Force	ξ_S	Record No., Location of Force	ξ_{SP}	Record No., Location of Force	ξ_H	Record No., Location of Force	ξ_W	Record No., Location of Force	ξ_{WP}
a) no modifications, unsupported end and edge beams	5.41		(0.4)**			135, A	0.91	146, E	0.37		
i) end and edge beams supported	6.4	76, E	0.37	76, E	0.91	77, A B C D	1.19 0.85 0.82 0.72				
ii) only end beams supported	6.38		(0.4)**			112, A B D	1.25 0.95 0.65	111, E	0.20	111, E	0.35
b) suspended ceiling with "lights"	6.20	92, E	0.71	92, E	0.72	93, A B C D	1.67 1.35 1.17 1.06	94, E	0.58	94, E	0.58
c) gypsum board ceiling	6.36	109, E	0.44	109, E		108, A B C D	1.30 1.18 1.07 0.96	106, E	0.40	106, E	0.42
d) steel-neoprene damping strips	7.8	115, E	1.0	115, E		114, A C	1.74 1.65	113, E	0.95	113, E	1.01

*Analysis bandwidth = 0.060 Hz.
**Extrapolated from comparisons between other cases of shaker impact and heel impact tests.

TABLE 7

Comparison of Measured and Calculated Initial Peak Acceleration Amplitudes from Heel Impacts

Floor Condition	Run No. and Location of Heel Impact	Measured Fundamental Frequency, Hz	Equivalent Inertial Mass, M_I, Eq. (4), kg	Equivalent Mass, M_J, Eq. (3), kg	Initial Peak Acceleration, a_0, % gravity		
					Calculated from Eq. (2)		
					Using M_I	Using M_J	Measured
End and Edge Beams Supported							
81 mm slab	33 A	6.57	6350	3540	4.69	7.18	4.72
	B				3.66		3.65
	C				3.50		3.65
	D				3.15		3.15
81 mm slab, transverse concrete beam	155 A	6.61	6860	4050	4.12	6.30	4.63
	B				3.21		4.02
	C				3.07		3.98
	D				2.76		3.77
106 mm slab, transverse concrete beam	190 A	6.25	8850	5190	3.03	4.65	3.41
	B				2.36		3.12
	C				2.27		3.03
	D				2.03		2.26
End and Edge Beams Unsupported							
81 mm slab	8 A	5.43	10700		2.18	5.9	6.00
	9 B						4.75
	10 C						4.85
	11 D						3.90

Total mass of 81 mm floor slab plus joists is 15,900 kg.

TABLE 8

Comparison of Measured and Calculated Fundamental Frequencies for Floor Sample

Floor Condition	Run No.	Fundamental Natural Frequency, Hz		
		Measured	Calculated	
End and Edge Beams Supported				
81 mm slab	33	6.57	6.83	
81 mm slab transverse concrete beam	159	6.75	6.83 (approx.)	
106 mm slab, transverse concrete beam	183	6.25	6.25	
End and Edge Beams Unsupported				
			Composite	Noncomposite
81 mm slab	8-11	5.43	4.7	5.5
81 mm slab, transverse concrete beam	147-150	5.6	4.8	5.6
106 mm slab, transverse concrete beam	166-180	5.3	4.3	5.2

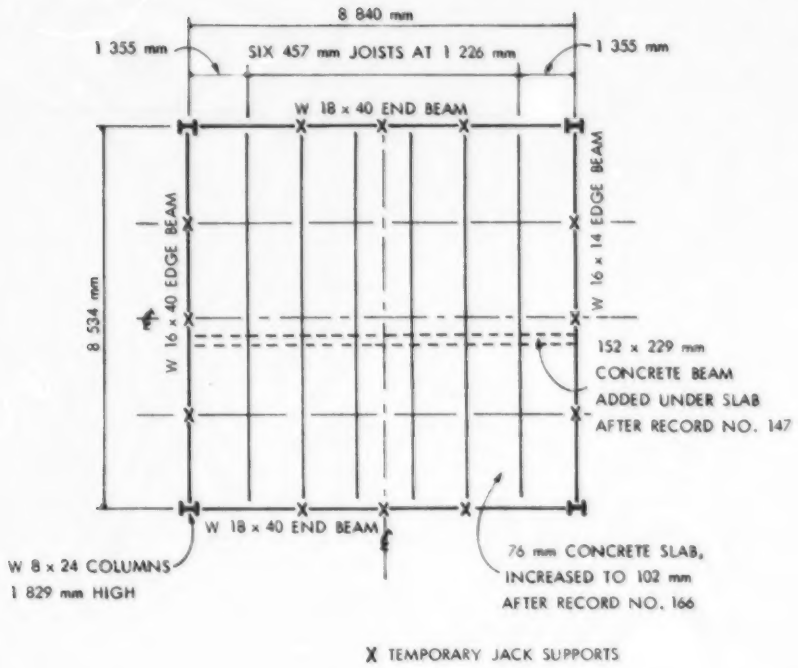

8 840 mm
1 355 mm
SIX 457 mm JOISTS AT 1 226 mm
1 355 mm
W 18 x 40 END BEAM
W 16 x 40 EDGE BEAM
W 16 x 14 EDGE BEAM
8 534 mm
152 x 229 mm
CONCRETE BEAM
ADDED UNDER SLAB
AFTER RECORD NO. 147
W 18 x 40 END BEAM
W 8 x 24 COLUMNS
1 829 mm HIGH
76 mm CONCRETE SLAB,
INCREASED TO 102 mm
AFTER RECORD NO. 166
X TEMPORARY JACK SUPPORTS

Figure 1. Layout of floor sample

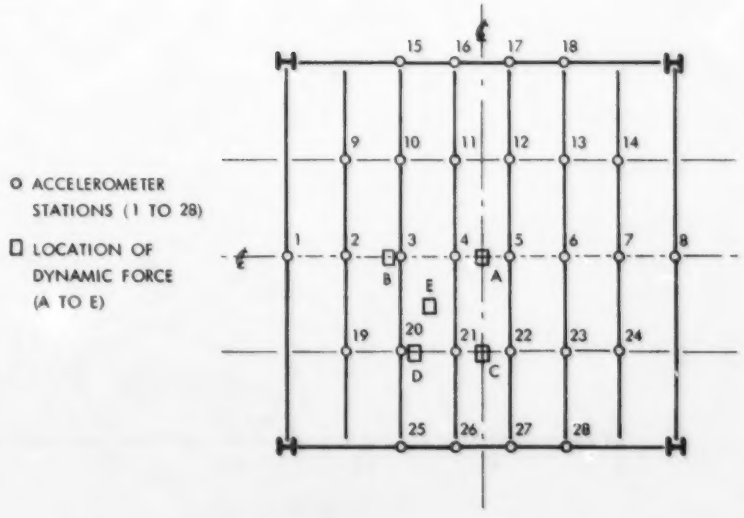


Figure 2. Locations of accelerometer stations (O) and dynamic force application (□)

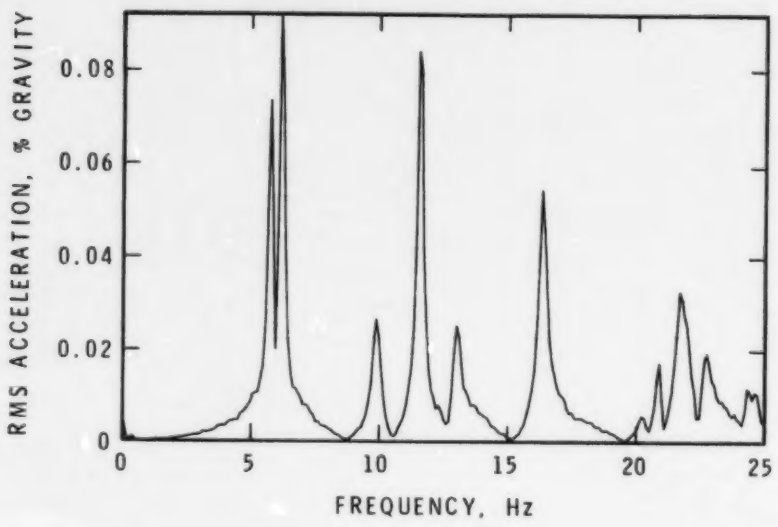


Figure 3. Fourier amplitude spectrum at location 21, run 118, shaker random excitation at location E, 81 mm concrete slab, steel-neoprene-steel sandwich strips, unsupported end and edge beams

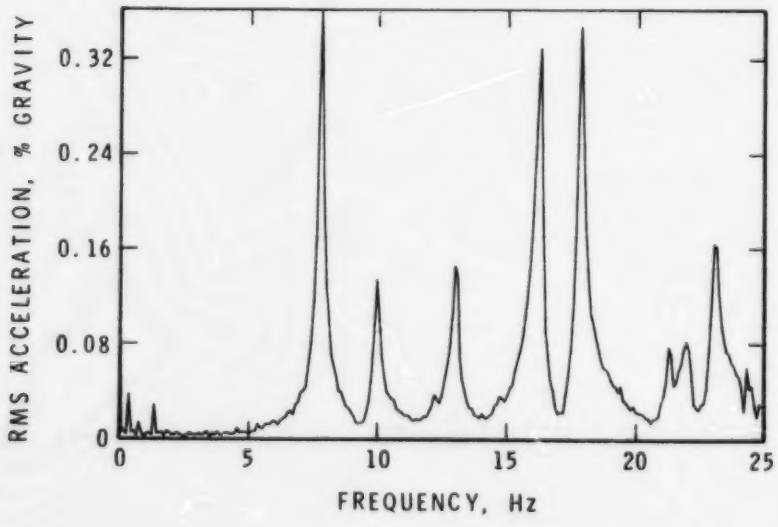


Figure 4. Fourier amplitude spectrum at location 21, run 115, shaker impact at location E, 81 mm concrete slab, steel-neoprene-steel sandwich strips, supported end and edge beams

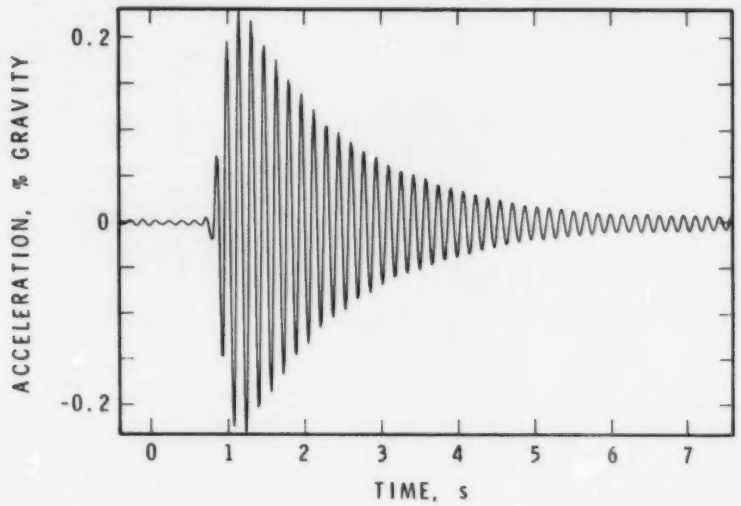


Figure 5. Free vibration decay at locations 4, 5, run 182, shaker impact at location A, 2 people standing at location A, 106 mm concrete slab with 152 × 229 mm transverse concrete beam, mode 1, unsupported end and edge beams

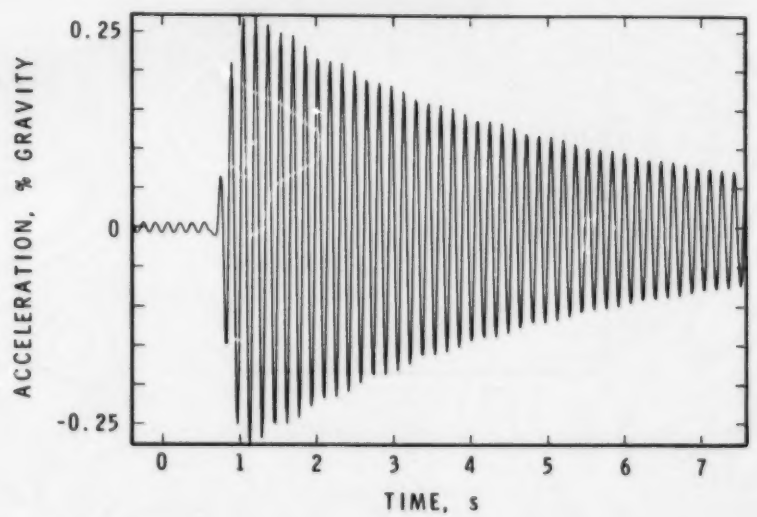


Figure 6. Free vibration decay at locations 4, 5, run 182, shaker impact at location A, mode 1, 106 mm concrete slab with 152 × 229 mm transverse concrete beam, unsupported end and edge beams

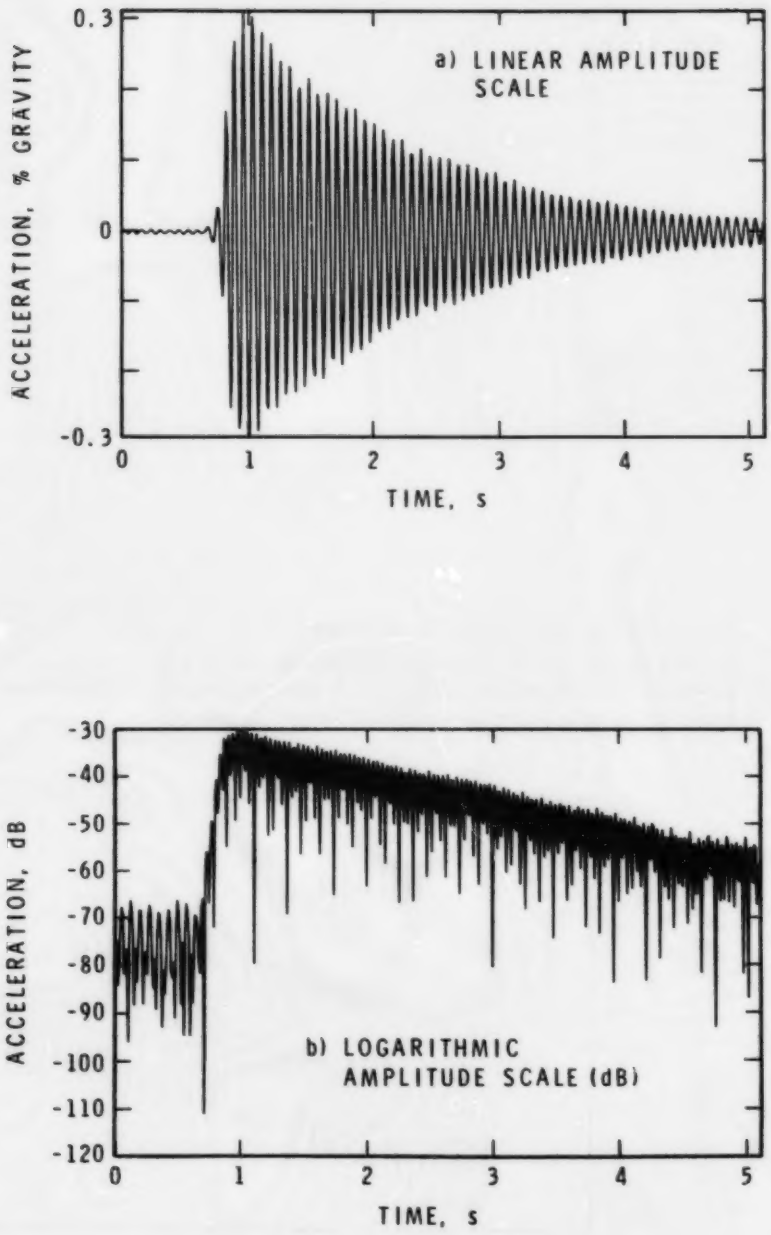


Figure 7. Free vibration decay at locations 3, 6, run 161, shaker impact at location B, mode 2, 81 mm concrete slab with 152 × 229 mm transverse concrete beam, unsupported end and edge beams

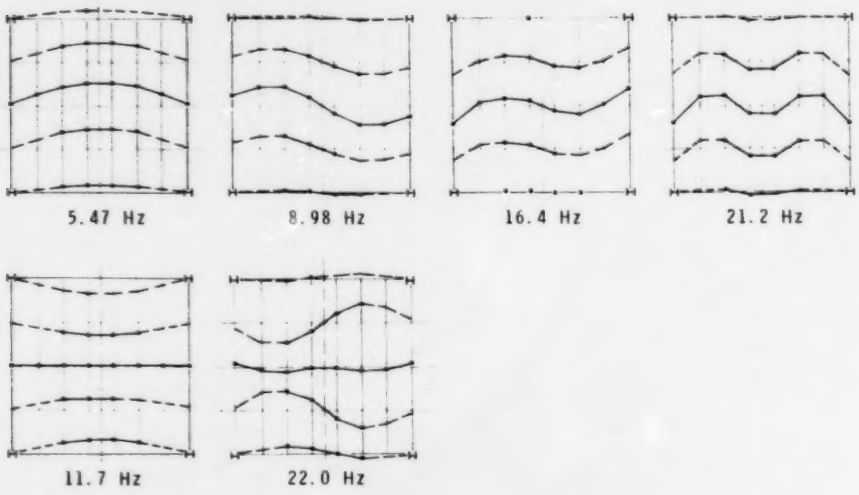


Figure 8. Mode shapes and natural frequencies of floor sample, run 5, shaker random excitation at location D, 81 mm concrete slab, unsupported end and edge beams

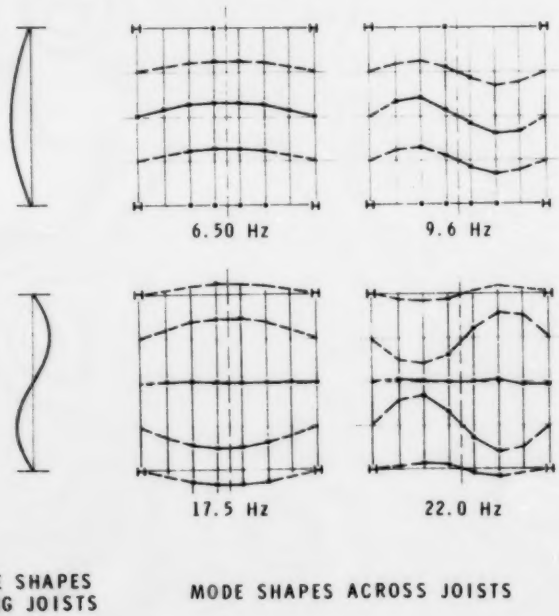


Figure 9. Mode shapes and natural frequencies of floor sample, run 29, shaker impact at location D, 81 mm concrete slab, unsupported end and edge beams

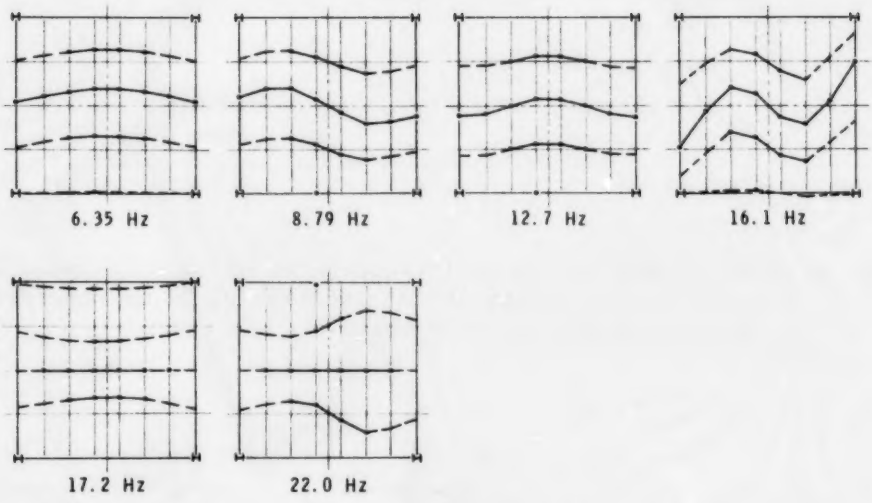
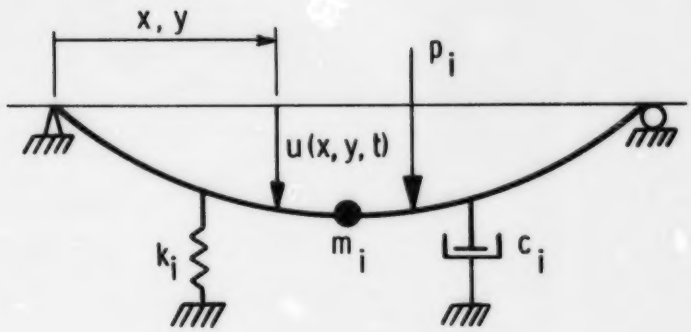


Figure 10. Mode shapes and natural frequencies of floor sample, run 111, shaker random excitation at location E, 81 mm concrete slab, supported end beams, unsupported edge beams



USING GENERALIZED COORDINATES, THE DISPLACEMENT IS

$$u(x, y, t) = \psi(x, y)\, z(t)$$

THE EQUATIONS OF MOTION ARE

$$\bar{M}\ddot{z} + \bar{C}\dot{z} + \bar{K}z = \bar{P}$$

WHERE $\psi(x, y)$ = MODE SHAPE

$z(t)$ = GENERALIZED COORDINATE

$\bar{M}$ = MODAL MASS = $\int \rho\psi^2\, dA + \sum m_i \psi_i^2$

$\bar{C}$ = MODAL DAMPING = $\int c\psi^2\, dA + \sum c_i \psi_i^2$

$\bar{K}$ = MODAL STIFFNESS = $\int EI(\psi'')^2\, dA + \sum k_i \psi_i^2$

$\bar{P}$ = MODAL FORCE = $\int \rho\psi\, dA + \sum p_i \psi_i$

AND A = AREA OF PLATE

EI = FLEXURAL STIFFNESS

Figure 11. Modal properties of beam or plate (Ref. 5)